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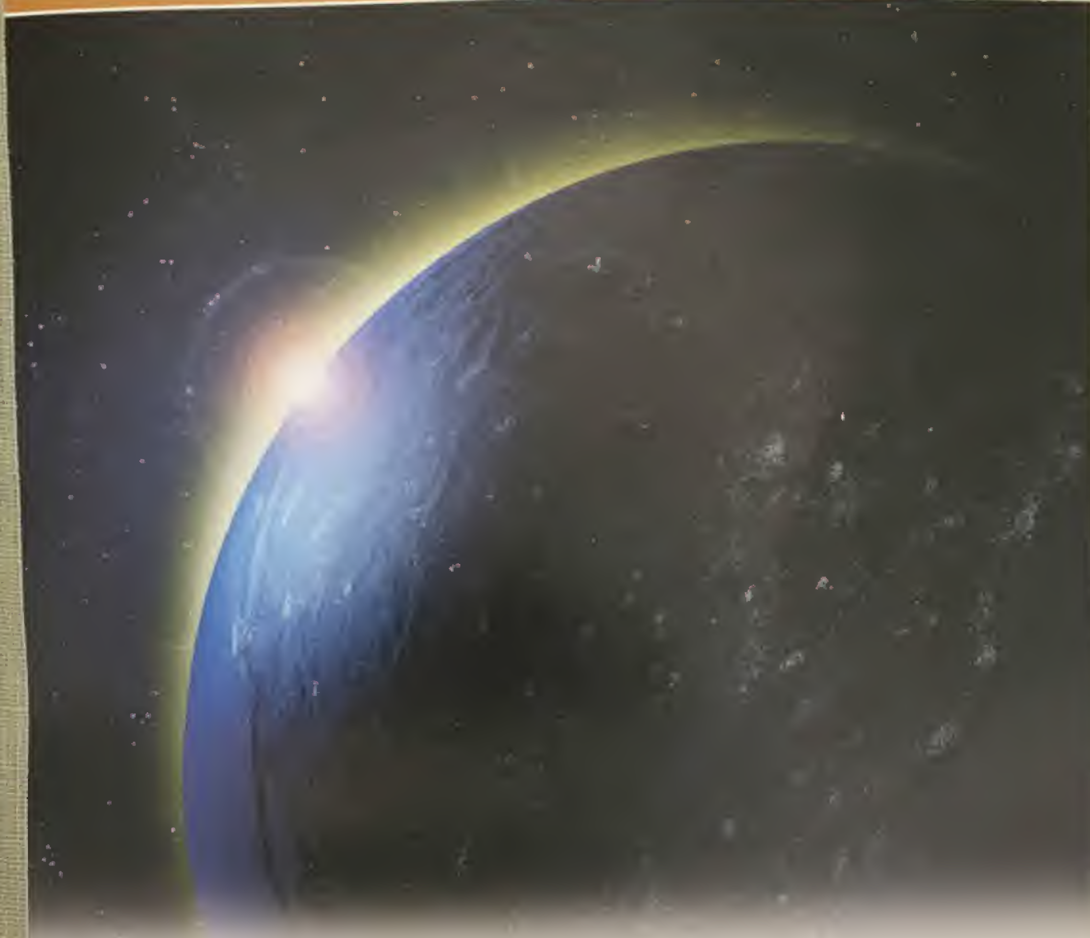

Algebra II

Taught by: Professor Murray H. Siegel
Sam Houston State University

Parts 1, 2, and 3

Course Guidebook

THE TEACHING COMPANY®

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Murray Siegel received his B.S. in Physics at the New York University College of Engineering in 1963 and his M.Ed. and Ed.S. (Math Ed.) from Georgia State University in 1975 and 1976. He received his Ph.D. (Math Ed.) from Georgia State University in 1978.

Dr. Siegel has accumulated a number of honors including membership in Tau Beta Pi (national engineering honor society); Kappa Delta Pi (national education honor society); 1986 Presidential Award for Excellence in Mathematics Teaching; Master Teacher—Woodrow Wilson National Fellowship Foundation; Who's Who in American Education 1987 – 1995; 1989 Star Teacher; 1990 – 91 Cobb County (GA) PAGE Educator of the Year; and the Kentucky Educational TV "Best Math Teacher in America" Award.

He is a member of the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics, the Mathematical Association of America, the Council of Presidential Awardees in Mathematics, the Mensa Mathematics Special Interest Group, the American Statistical Association, and the American Mathematical Association of Two-Year Colleges. He is a frequent speaker at regional and national conventions of these organizations.

Prior to assuming his position at Sam Houston State University, where he teaches future math teachers, Dr. Siegel taught at the J.L. Mann Academy for Mathematics, Science and Technology in Greenville, South Carolina.

Dr. Siegel's experience outside of public education includes Project Manager, Behavioral Research Laboratories; Executive Vice-President for Operations, Henderson Few & Co.; Department Manager, Goodbody & Co., a Wall Street brokerage firm; Food Production Manger, Marriott Corp.; and Captain, USAF.

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Algebra II
Parts I, II, III

Lesson One
Introduction

I. Rationale for learning Algebra II

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- **B.** models to predict what will happen

II. Equations

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- **B.** solving systems of equations
- **C.** types of equations
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  2. quadratic
  3. polynomial
  4. rational
  5. exponential
  6. logarithmic

III. Visual examples of the use of Algebra II

- **A.** scatter plots
- **B.** fit lines
- **C.** intersection of lines

IV. How to use this series

- **A.** watch the tapes
- **B.** stop the tape, rewind and review
- **C.** work problems
- **D.** find similar problems in your text and work those

V. Where do we go after Algebra II

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   C. looking at points on the line

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   D. parallel lines have the same slope
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   C. write the equation
   D. check using the other point

IV. Finding the equation of a line if given one point on the line and the equation of a parallel line
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   B. the slope of the unknown line is the same as the parallel line
   C. use the slope and the coordinates of the given point to solve for the y-intercept
   D. write the equation
   E. check

V. Finding the equation of a line if given one point on the line and the equation of a perpendicular line
   A. get the perpendicular line's equation in slope intercept form
   B. the slope of the unknown line is the negative reciprocal of the perpendicular line
   C. use the slope and the coordinates of the given point to solve for the y-intercept
   D. write the equation
   E. check

VI. Summary

Lesson Eight
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I. Plotting points
- **A.** using ordered pairs to plot points
- **B.** finding points using the grid of a graph

II. Using a roster of points
- **A.** given an equation, write the equation in slope intercept form
- **B.** select various values of x for the roster
- **C.** substitute each value of x into the equation and solve for the corresponding value of x
- **D.** list ordered pairs of x,y in the roster
- **E.** plot points from the roster
- **F.** check that points line up
- **G.** draw the line

III. Graphing a line using the intercepts
- **A.** get the equation in $ax + by = c$ form
- **B.** substitute zero for x and get the y-intercept
- **C.** plot the y-intercept
- **E.** substitute zero for y and get the x-intercept
- **F.** plot the X-intercept
- **G.** connect the two intercepts with a line

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- **A.** write the equation in slope intercept form
- **B.** plot the y-intercept
- **C.** starting at the y-intercept, move one to the right and use the slope to plot the change in y
- **D.** continue to use the slope to plot more points
- **E.** connect the points with a line

V. Graphing using the point slope form
- **A.** $y - y_1 = m(x - x_1)$ is the point slope form, where x_1 and y_1 are the x and y coordinates of a point on the line
- **B.** plot the point x_1, y_1
- **C.** use the slope to plot other points on the line
- **D.** connect the points with a line

Lesson Nine
Graphs of Linear Inequalities

I. Write the inequality in proper form
- **A.** solve the inequality for y
- **B.** the inequality is in slope intercept form

II. Sketch the line
- **A.** using the slope intercept form, sketch the line of the equation $y = mx + b$ (the line should be lightly sketched in)
- **B.** if the inequality does not contain equals (it is a strictly less than or strictly greater than situation), sketch the line as a dashed line
- **C.** if the inequality does contain equals, sketch the line as a solid line

III. Shading the inequality
- **A.** if the inequality in slope intercept form contains less than (either less than or equal to, or strictly less than) shade the region below the line
- **B.** if the inequality in slope intercept form contains greater than, shade above the line

IV. Checking
- **A.** select a point that is in the shaded region
- **B.** replace the x and y in the original inequality with the coordinates of the selected point—the inequality should work for these values
- **C.** select a point that is outside the shaded region
- **D.** replace the x and y in the original inequality with the coordinates of the selected point—the inequality should not work for these values

V. The special case of an inequality involving only x
- **A.** sketch the vertical line of the equation $x = c$
- **B.** use dashes or a solid line as before
- **C.** shade to the right if the inequality contains greater than
- **D.** shade to the left if the inequality contains less than

VI. The special case of an inequality involving only y
- **A.** sketch the horizontal line of the equation $y = c$
- **B.** use dashes or a solid line as before
- **C.** shade above the line if the inequality contains greater than
- **D.** shade below the line if the inequality contains less than

Lesson Ten

Solving Systems of Two Linear Equations

I. Rational for solving systems of linear equations
   A. look at a graph of a system picturing the supply and demand for food
   B. the intersection point is the solution to the system

II. Solving by graphing
   A. write the two equations in slope intercept form
   B. graph the equations
   C. find the coordinates of the intersection point
   D. check the values of x and y in both original equations

III. Problems with using the graphing method
   A. equations may be difficult to graph
   B. the intersection points may involve fractions which would be difficult to evaluate on graph paper

IV. Solving by substitution
   A. get one of the equations to show x as a function of y or y as a function of x
   B. if y is a function of x, substitute that function for y in the second equation
   C. you now have one equation with one variable
   D. solve the equation for the one variable
   E. substitute the solution to solve for the second variable
   F. check both original equations

V. Practice problem
   A. solve the system using graphing
   B. solve the system using substitution
   C. check your answers with the answers on the tape

VI. Problem with substitution
   A. you may be forced to use fractional coefficients which can make the solution "messy"

VII. Solving by the use of addition
   A. get the two equations into $ax + by = c$ form
   B. if one of the variables has coefficients that are the same number with opposite signs, add the two equations
   C. you are left with one equation
   D. solve as you did in the substitution method

Lesson Eleven

Solving Systems of Two Linear Equations Using Elimination

I. Problem with solving using addition
   A. you do not always have coefficients that are additive opposites

II. Solution where only one equation needs to be multiplied
   A. multiply one equation so that one variable has a coefficient that is the additive opposite of the coefficient of that variable in the second equation
   B. add the new equation (after multiplication) to the second equation
   C. solve as was done in the addition method
   D. check in both original equations

III. Multiplying both equations, without a sign change
   A. find a variable whose coefficients have opposite signs
   B. multiply both equations so that the selected variable has coefficients that are additive opposites
   C. add the two new equations to eliminate the selected variable
   D. solve
   E. check in both original equations

IV. Multiplying both equations with a sign change
   A. select a variable to be eliminated
   B. multiply both equations so that the coefficients of the selected variable are additive opposites
   C. add the two new equations to eliminate the selected variable
   D. solve
   E. check in both original equations

Lesson Twelve

Solving Systems of Three Linear Equations And Systems of Linear Inequalities

I. Solving three equations with three variables
   A. select a variable to eliminate
   B. select one pairing of equations from the three equations
   C. multiply the two equations so that the coefficients of the selected variable are additive opposites
   D. add the two new equations to eliminate the selected variable
   E. select a second pairing of equations from the original three equations
   F. eliminate the selected variable by multiplying and addition
   G. you now have two equations with two variables
   H. solve for the two variables using the elimination method
   I. substitute the values for the two variables into one of the original equations and solve for the third variable
   J. check in the remaining two original equations

II. Solving systems of linear inequalities
   A. using the method for graphing linear inequalities, sketch and shade the graph for one of the inequalities
   B. using a different color or method of shading, sketch and shade the second inequality
   C. continue to sketch and shade the remaining inequalities, using a different color or method of shading for each inequality
   D. the region that contains all the different types or colors of shading contains all the points that satisfy all the inequalities
   E. this region is the solution to the system of inequalities
   F. identify the coordinates of the intersection points where the lines cross

Lesson Thirteen

Functions

I. The definition of function
   A. a relation where each item of input is related to only one item of output
   B. definition and examples of domain
   C. definition and examples of range
   D. using "potato" diagrams
   E. examples of functions and non-functions

II. Methods of defining a function
   A. diagram
   B. ordered pairs
   C. rule
   D. graph

III. The inverse
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   B. finding the inverse by solving for x
   C. finding the inverse of a linear function
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   C. reciprocal
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Lesson Fourteen
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I. The quadratic function
   A. general form
   B. specific examples

II. Graphing the quadratic function $y = ax^2$
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   B. rosters for different coefficients
   C. the parabola
   D. changes in the graph related to different coefficients
   E. the graph if the coefficient of the squared term is negative

III. Graphing $y = ax^2 + c$
   A. the parabola if the lead coefficient is one and the constant changes
   B. the parabola if the lead coefficient is negative

IV. The graph of the general quadratic function
   A. creating a roster of points
   B. the vertex of the parabola
   C. finding the turning point at (h,k)
   D. finding the x value of the turning point using the coefficients a and b
   E. finding the Y value of the turning point
   F. plot the vertex
   G. plot the y-intercept at (0,c)
   H. plot the point symmetric to the y-intercept
   I. sketch the parabola

V. Sketching quadratic inequalities
   A. sketch the parabola for the equation (draw sketch lightly)
   B. if the inequality contains "equals", darken in the parabola
   C. if the inequality does not contain "equals", use dashes for the parabola
   D. if the inequality contains "less than", shade below the parabola
   E. if the inequality contains "greater than", shade above the parabola

Lesson Fifteen
Solving Quadratic Equations

I. Solving $ax^2 = c$
   A. divide both sides of the equation by the number a
   B. take the square root of both sides of the equation
   C. there are two answers—one positive, the other negative

II. Solving by factoring
   A. get the equation in the form of a quadratic equal to zero
   B. use factoring methods to completely factor the quadratic
   C. set each factor equal to zero
   D. solve
   E. check both solutions in the original equation

III. Problem with the factoring method
   A. there are many quadratics that cannot be factored

IV. Solving by completing the square
   A. write the equation in the form $ax^2 + bx = c$
   B. examine how the left side of the equation could be part of a perfect square trinomial
   C. divide by two
   D. square the number obtained
   E. add this number (after squaring) to both sides of the equation
   F. write the left side of the equation as the square of a binomial
   G. take the square root of both sides of the equation (remember that there are two answers to the square root of a number, one positive and the other negative)
   H. solve for x (the solutions may contain a square root)

V. Completing the square if b is an odd number
   A. follow the same steps but note that b divided by two is a fraction with two in the denominator
   B. the square of this number will have four in its denominator

VI. Completing the square if the lead coefficient is not equal to one
   A. isolate the linear and quadratic terms as was done previously
   B. divide both sides of the equation by the lead coefficient
   C. continue the solution as was previously done

Lesson Sixteen
The Quadratic Formula

I. The quadratic formula
   A. start with the general quadratic equation
   B. solve for x by completing the square
   C. the result is the general solution for all quadratics, called the quadratic formula

II. The discriminant
   A. $b^2 - 4ac$ is the discriminant since it is inside a square root symbol
   B. if the discriminant is positive, there are two real roots (root is another name for answer or solution)
   C. if the discriminant is zero, there is only one real root
   D. if the discriminant is negative, there are no real roots

III. Steps in solving the quadratic
   A. get the equation in standard form
   B. define the values of a, b and c
   C. find the discriminant (if negative, there are no real roots)
   D. solve for x using the quadratic formula
   E. check

IV. Examples of solutions using the quadratic formula
   A. equation with two real roots
   B. equation with one real root
   C. equation with no real roots
   D. equation with two fractional roots

V. Using the quadratic formula to sketch parabolas
   A. find and plot the vertex
   B. find and plot the Y-intercept
   C. find and plot the point symmetric to the Y-intercept
   D. use the quadratic formula to find the X-intercepts
   E. plot the X-intercepts
   F. sketch the parabola

Lesson Seventeen
Imaginary Numbers

I. Introduction to i
   A. the need for a symbol for the square root of a negative—the negative discriminant
   B. i is defined as the square root of negative one
   C. the square if i is negative one
   D. the square root of negative four is 2i
   E. the use of the square root of negative one—electricity
   F. using i to write solutions to quadratic equations where the discriminant is negative

II. Arithmetic of complex numbers
   A. a complex number is of the form a + bi
   B. examples of complex numbers
   C. demonstrating that a real number and an imaginary number are each examples of a complex number
   D. adding complex numbers by adding the real parts and then adding the imaginary parts
   E. using a similar technique to subtract complex numbers
   F. multiplying an imaginary number by an imaginary number
   G. multiplying a complex number by a complex number
   H. multiplying a number by its conjugate (the conjugate of a + bi is a - bi)
   I. using the conjugate to make a complex denominator a real number
   J. multiplying the numerator and denominator of a ratio of complex numbers by the conjugate of the denominator to solve a division problem

III. The complex plane
   A. the X-axis is for the real part of a complex number (a)
   B. the Y-axis is for the coefficient of the imaginary part of a complex number (b)
   C. plotting points that represent complex numbers

IV. Solving quadratics with negative discriminants
   A. solving for x
   B. checking using complex arithmetic
   C. if a + bi checks as the solution to a quadratic then its conjugate, a - bi, is also a solution (the conjugate need not be checked)

V. Complex cube roots of real numbers

Lesson Eighteen
Quadratic and Rational Inequalities

I. Quadratic inequalities greater than zero
   A. factor the quadratic
   B. if the product of the factors is positive then both factors are positive or both factors are negative
   C. solve the two systems of inequalities
      1. if a solution has two inequalities that are both in the same direction then the simple way to write the solution is the inequality that is further in that direction (example $x > 2$ and $x > 5$ can be written more simply as $x > 5$)
      2. check the solution by substituting values that should work and those that should not work into the original inequality

II. Quadratic inequalities less than zero
   A. factor the quadratic
      1. if the product of the factors is negative then one factor must be positive and the other must be negative
   B. solve the two systems of inequalities
      1. check using numbers that should work and numbers that should not work in the original inequality

III. Rational inequalities
   A. if "equals" is included in the inequality, the numerator can be zero but the denominator cannot be zero
   B. if the inequality contains "greater than", the numerator and denominator must both be positive or both be negative
   C. if the inequality contains "less than", the numerator and denominator must have opposite signs
   D. solve the two systems of inequalities
   E. check

IV. Alternative method of solution
   A. write the inequality as an equation equal to zero
   B. solve the equation by setting factors (for quadratic) or numerator and denominator (for rational) equal to zero
   C. solutions represent endpoints of regions on the number line
   D. test one number from within each region defined by the endpoints (do not use the endpoints values) in the original inequality
   E. if the test point works, that region is part of the solution
   F. if the test point does not work, that region is not part of the solution

Lesson Nineteen
Polynomial Division

I. Dividing a polynomial by a binomial
   A. use the long division method from arithmetic
   B. check the answer to the division problem by substituting ten for x

II. Develop the synthetic division method
   A. when dividing by x - a, use the number a for synthetic division
   B. write down all coefficients of the polynomial to be divided
   C. bring down the first coefficient
   D. multiply this number by the divisor (a) and subtract the product from the next coefficient
   E. continue this process
   F. the bottom line contains the coefficients of the answer polynomial with the last number being the remainder
   G. write the answer as a polynomial with the remainder written as a fraction; the remainder number in the numerator and x - a in the denominator
   H. if the polynomial to be divided has missing places (e.g. $2x^3 - 4x + 3$ is missing an x^2 term) be sure to write a zero for the coefficient(s) of the missing term(s)

III. Finding a factor of a polynomial
   A. if the remainder is zero then x - a is a factor of the polynomial

IV. Dividing by bx - a when b is other than one
   A. divide bx - a by b
   B. divide the polynomial to be divided by b
   C. synthetically divide the new polynomial by the number a/b
   D. write the remainder without a fraction in the denominator

Lesson Twenty
Zeroes of a Polynomial

I. Rationale for solving for the zeroes of a function

A. to sketch a function it is helpful to have the X-intercepts since the X-intercepts are when y is zero, the zeroes of a polynomial are the X-intercepts

II. Factor Theorem

A. if x - a is a factor of a polynomial then x = a is a zero of the function. This is demonstrated using our knowledge of multiplying polynomials and solving a factored equation equal to zero

III. Rational zeroes of a polynomial

A. if x - a, x - b and x - c are the factors of a polynomial then a times b times c must equal the constant term of the polynomial

B. if dx - a, ex - b and fx - c are the factors of a polynomial then d times e times f must equal the value of the lead coefficient of the polynomial

C. if a zero of a polynomial is rational, its numerator must be a factor of the constant term of the polynomial and its denominator must be a factor of the lead coefficient

D. if the lead coefficient of a polynomial is 5 and its constant term is 6, then the only rational zeros are 1, 2, 3, 6, 1/5, 2/5, 3/5, 6/5 and their negatives

IV. Solving for the zeroes of a polynomial

A. list all possible rational zeroes

B. use synthetic division to test possible rational solutions

C. once you have used synthetic division to reduce the remaining polynomial to a quadratic, use the quadratic formula to solve for the remaining two zeroes which may be irrational or complex

D. check

E. if you are asked to find the factors of a polynomial, use the zeroes to write the factors (if -3/4 is a zero then $4x + 3$ is a factor)

V. The X-intercepts of a polynomial

A. the zeroes of a polynomial are the X-intercepts

B. once the zeroes are found, they can be plotted to help in the sketch of the polynomial function

Lesson Twenty-One
Sketching Polynomials

I. Intercepts

A. Y-intercept—constant term

B. plot Y-intercept

C. X-intercepts—zeros of the polynomial

D. plot X-intercept

II. End behavior

A. what happens to the function as x becomes a very large positive number or a very large negative number

DEGREE	LEAD COEFFICIENT	LEFT SIDE	RIGHT SIDE
even	+	up	up
even	-	down	down
odd	+	down	up
odd	-	up	down

B. sketch in end behavior

II. Turning points

A. the maximum number of turning points is always one less than the degree

B. if the number of turning points is less than the maximum it MUST be an even number less than the maximum (e.g. if the degree is 5, the polynomial can have four, two or no turning points

C. given Y-intercept, X-intercepts and end behavior; determine the number of turning points

D. sketch the curve

IV. Zeroes that repeat

A. multiplicity—a zero that occurs more than once (e.g. the zeroes of $y = x^3 - 10x^2 + 25x$ are 0, 5 and 5; thus, 5 has a multiplicity of two

B. examine the sketch of this function and note that at $x = 5$ there is a zero and a turning point

V. Imaginary zeros

A. what happens if the function has two imaginary or complex zeroes?

B. imaginary and complex numbers are not found on the X-axis

C. only real zeroes are X-intercepts

Lesson Twenty-Two
Sketching Rational Functions

I. Intercepts
   A. Y-intercept occurs when x is zero
   B. X-intercepts are values of x that make the numerator of the function zero while the denominator is not zero
   C. plot intercepts

II. Vertical asymptotes
   A. values of x that make the denominator zero are not in the domain of the function
   B. examine values of x close to those values that cannot be in the domain
   C. draw vertical asymptote as a dashed line

III. Horizontal asymptotes
   A. examine end behavior in three situations
   B. numerator's degree > denominator's degree—function goes to positive or negative infinity
   C. numerator's degree = denominator's degree—function has a horizontal asymptote given by y = ratio of lead coefficients
   D. numerator's degree < denominator's degree—function has a horizontal asymptote that is the X-axis
   E. sketch asymptote as a dashed horizontal line or show infinite end behavior

IV. Sketching the graph
   A. use intercepts, vertical asymptotes and horizontal asymptotes or infinite end behavior to sketch the function
   B. if this information is insufficient, find values of the function for selected values of x and plot these points

V. Holes
   A. examine a rational function where the numerator and denominator have a common factor
   B. examine this function for values of x close to the value that makes the common factor equal to zero
   C. there is a missing point at the value of x that causes the common factor to be zero

Lesson Twenty-Three
Square Roots and Cube Roots

I. Square root equations
   A. square both sides and solve
   B. check all solutions in the original equation
   C. it is likely in a square root equation to have solutions that do not work in the original equation
   D. always isolate the square root before squaring both sides

II. Graphing square root functions
   A. examine y = square root of x. Notice the end point
   B. Y-intercept
   C. X-intercepts—set function equal to zero and solve
   D. end point—value of x that causes term inside the square root to equal zero
   E. plot intercepts and end point
   F. plot additional points if you are unsure of the shape of the graph
   G. sketch graph
   H. an example whose graph is a semicircle

III. Cube root equations
   A. cube both sides of the equation
   B. solve
   C. check
   D. always isolate the cube root before cubing both sides of the equation

IV. Sketching cube root functions
   A. examine the graph of y = cube root of x. Notice the point where the curvature changes
   B. Y-intercept
   C. X-intercepts
   D. zero point—the point where the curvature changes; this is the value of x that causes the term inside the cube root to be zero
   E. sketch the graph

Lesson Twenty-Four
Exponential Functions

I. Importance of exponential functions
   A. national debt
   B. population growth
   C. learning curve

II. Operations with exponents
   A. rule of multiplication: if the bases are the same, keep the base and add the exponents
   B. rule of division: if the bases are the same, keep the base and subtract the exponents
   C. rule of raising a power to a power: keep the base and multiply the powers
   D. any base to the zero power is one
   E. a negative exponent represents the reciprocal

III. Sketching a exponential function
   A. create a roster of points for $y = 2^x$
   B. sketch the graph
   C. note that y cannot be zero or negative
   D. show that $y = 2^{-x}$ is the same as $y = 1/2^x$
   E. create a roster of points for this function
   F. sketch the function
   G. compare sketch to the first sketch
   H. create a roster for $y = 3(1/2)^x$
   I. sketch the graph
   J. relate this graph to half life

IV. Solving exponential equations
   A. write the equation so that each side uses the same base (e.g. $3^x = 81$ can be rewritten as $3^x = 3^4$)
   B. if the two bases are the same, set the exponential terms equal and solve this equation
   C. check
   D. remember that a fraction such as 1/27 can be written as 3 to the negative third power
   E. in some cases a calculator may be needed to check the original equation

Lesson Twenty-Five
Logarithmic Functions

I. Inverses
   A. examine inverse operations
   B. develop the idea that the logarithmic function is the inverse of the exponential function
   C. any logarithmic equation can be written as an exponential equation
   D. solve simple logarithmic equations by converting the equation to its exponential form (the variable may be the base, the number or the logarithm)

II. Logarithmic notation
   A. $\log_b$ means the logarithm using the number b as a base
   B. log indicates the logarithm base ten

III. Operations with logarithms
   A. all rules are based on our knowledge of exponents
   B. the logarithm of the product is the sum of the logarithms
   C. the logarithm of the quotient is the difference of the logarithms
   D. the logarithm of a number raised to a power is the power multiplied by the logarithm of the number

IV. Solving logarithmic equations
   A. use the rules of operating with logarithms to simplify the equation so that it has only one logarithm in it
   B. convert the logarithmic equation to an exponential equation
   C. solve the exponential equation
   D. check all solutions in the original logarithmic equation
   E. to find the $\log_b a$ on a calculator compute log a divided by log a

V. Sketching logarithmic functions
   A. to sketch $y = \log_2 x$ convert the equation to $x = 2^y$
   B. create a roster of points by selecting values of y and solving the exponential form for x
   C. plot points
   D. notice that the function is asymptotic to the Y-axis (you cannot take the log of zero or of a negative number)

Lesson Twenty-Six
Matrices and Determinants

I. Introduction to matrices
   A. rectangular array: an example
   B. demonstrate rows and columns
   C. define the meaning of the order of a matrix

II. Adding and subtracting matrices
   A. adding a 2 by 3 to a 2 by 3 matrix
   B. subtracting a 4 by 2 from a 4 by 2 matrix
   C. repeated addition of matrices
   D. using scalar multiplication to simplify repeated addition

III. Multiplying matrices
   A. multiplying the first row of the first matrix by the first column of the second matrix to obtain the element in the first row, first column location in the answer matrix
   B. completing the matrix multiplication
   C. developing the idea that to multiply two matrices, the number of columns in the first matrix must be the same as the number of rows in the second matrix
   D. the number of rows in the answer will equal the number of rows in the first matrix and the number of columns in the answer will equal the number of columns in the second matrix
   E. multiplying square matrices
   F. demonstrating that multiplication of matrices is not commutative

IV. Determinants
   A. finding the determinant of a 2 by 2 matrix
   B. a method to compute the determinant of a 3 by 3 matrix

V. The inverse of a matrix
   A. define the method to find the inverse of a specific 2 by 2 matrix
   B. generalize the method for finding the inverse of a 2 by 2 matrix
   C. multiply a matrix by its inverse
   D. discuss the identity matrix
   E. the inverse will be used in the next lesson to solve systems of equations

Lesson Twenty-Seven
Solving Systems Of Equations Using Matrices

I. Solving using the inverse
   A. write a system of two equations
   B. write the same system as a matrix equation—the matrix of the coefficients multiplied by the column matrix containing X and Y is set equal to a column matrix containing the numbers on the right side of the equals symbol in the original equations
   C. multiply both sides of the matrix equation by the inverse of the coefficient matrix
   D. find the inverse of the coefficient matrix
   E. the inverse times the coefficient matrix times the X Y column matrix equals the X Y matrix
   F. X equals the number in the first row, first column of the matrix on the right side of the equation
   G. Y equals the number in the second row, first column of the matrix on the right side of the equation

II. Solving using determinants
   A. given a system of two equations with two variables, find the determinant of the coefficient matrix (this number is D)
   B. replace the first column of the coefficient matrix with the numbers to the right of the equals sign in the respective equations
   C. find the determinant of this matrix
   D. X equals the quotient of this determinant divided by D
   E. replace the second column of the coefficient matrix with the numbers to the right of the equals sign in the respective equations
   F. find the determinant of this matrix
   G. Y equals the quotient of this determinant divided by D
   H. this method is called Cramer's Rule
   I. solve a system of three equations with three variables using Cramer's Rule

Lesson Twenty-Eight
Recursive Functions

I. Introduction to recursive functions
  A. an example of a recursive function
  B. the domain is the whole or counting numbers
  C. the first term is given and a rule for finding succeeding terms is provided
  D. make a roster of points for a sequence of values for n (the domain variable)

II. Recursive functions that are linear
  A. create a roster of points for the given recursive function
  B. compute the difference between each term and the next term
  C. if all the differences are the same, the function is linear
  D. graph the points and notice how they line up
  E. select two points from the roster
  F. find the slope by dividing the difference in y by the difference in x
  G. use the slope and the X and Y from one point to solve for the Y-intercept
  H. write the linear function as a function of n
  I. check using another point from the roster

III. Recursive functions that are quadratic
  A. create a roster of points for the given recursive function
  B. plot the points and note that the points do not line up
  C. examine the differences between values of the function; they are not the same
  D. compute the second differences, which is found by computing the difference between succeeding differences
  E. the second differences are all the same; this indicates a quadratic function
    1. the quadratic function in n is written $a_n = an^2 + bn + c$
    2. select three points from the roster and substitute the values for n and a_n
    3. you now have three equations with three variables— a, b and c
    4. solve the systems for a, b and c
    5. write the function as a quadratic equation in n
    6. check the equation using a fourth point from the roster
    7. work another similar problem where a and b are fractions

Lesson Twenty-Nine
Sequences and Series

I. Summation notation
  A. introduce the use of S (sigma)
  B. work a summation problem

II. Arithmetic sequences
  A. definition and examples of the arithmetic sequence—a sequence of numbers with a common difference
  B. define "a_1" as the first term of the sequence
  C. define "a_n" as the nth term of the sequence
  D. define "d" as the common difference
  E. write how each succeeding term would be computed
  F. generalize to show that the nth term would be the first term plus (n - 1) times d
  G. write the formula for the nth term
  H. use the formula to solve a problem

III. Arithmetic series
  A. a series is the sum of the terms of a sequence
  B. demonstrate by pairing terms of an arithmetic sequence that the sum of n terms is found using $(n/2)(a_1 + a_n)$
  C. use this method to find the sum of a given arithmetic sequence

IV. Geometric sequences
  A. definition and examples of the geometric sequence—a sequence of numbers with a common multiplier or ratio
  B. verify that $a_n = a_1 (r^{n-1})$ for a geometric sequence
  C. use this formula to find the nth term of a given geometric sequence

V. Geometric series
  A. derive the formula for the sum of a geometric sequence
  B. if the common ratio has an absolute value between zero and one, and the geometric sequence has an infinite number of terms, the sum can be found using the formula $a_1/(1 - r)$
  C. use the formula to find the sum of a given infinite geometric sequence

Lesson Thirty
Introduction To Trigonometry

I. The rationale for trigonometry

- **A.** the use of similar right triangles to measure distances
- **B.** the names for the three sides of a right triangle—opposite, adjacent and hypotenuse

II. Trigonometric ratios

- **A.** using the half-chord of a circle of unit radius, define the ratio of the opposite side to the hypotenuse
- **B.** due to an error in translation from Arabic to Latin, this ratio is called the <u>sine</u>
- **C.** demonstrate that the since of the complementary angle is the adjacent over the hypotenuse for the first angle
- **D.** the ratio of the adjacent to hypotenuse is called the <u>cosine</u>
- **F.** using the tangent to a unit circle, define the ratio of the opposite side to the adjacent
- **G.** the ratio of opposite to adjacent is called the <u>tangent</u>

III. Trigonometric functions for various angles

- **A.** examine what happens to the sides of a right triangle as you change the angle
- **B.** use these trends to find the sine, cosine and tangents of zero degrees and ninety degrees
- **C.** use the calculator to find the sine, cosine and tangent for various angles

IV. Solving problems using trigonometry

- **A.** given a specific problem where an angle and the side opposite the angle of a right triangle are known, find the adjacent side using tangent
- **B.** for the same problem find the hypotenuse using sine
- **C.** check the answers for reasonableness
- **D.** given the opposite and adjacent sides of a right triangle, find one angle using tangent
- **E.** for the same problem find the hypotenuse using sine
- **F.** check by using the Pythagorean Theorem to check that the three sides form a right triangle

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Guidebook Contents

Parts 1, 2, and 3

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